JEE MAINS-2016

09-04-2016 (Online)

IMPORTANT INSTRUCTIONS

1. The test is of **3** hours duration.
2. The Test Booklet consists of **90** questions. The maximum marks are **360**.
3. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
4. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

PART-A-PHYSICS

1. In the following 'I' refers to current and other symbols have their usual aning. Choose the option that corresponds to the dimensions of electrical conductivity :

(1) $ML^{-3}\,T^{-3}\,I^{2}$ (2) $M^{-1}\,L^{3}\,T^{3}\,I$ (3*) $M^{-1}\,L^{-3}\,T^{3}\,I^{2}$ (4) $M^{-1}\,L^{-3}\,T^{3}\,I$

Sol. $\vec{J} = \sigma\vec{E}$

$\vec{J} = I$

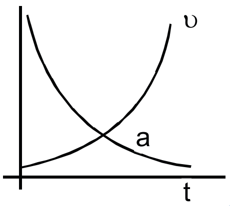
$[\vec{J}] = IL^{-2}$

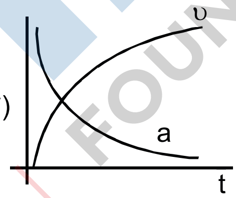
$[\vec{E}] = \left[\frac{F}{q}\right] = \frac{[F]}{[q]} = \frac{MLT^{-2}}{IT} = MI^{-1}LT^{-3}$

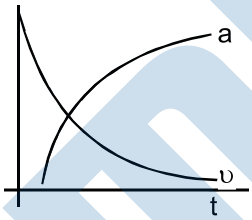
$IL^{-2} = [\sigma]\ MI^{-1}LT^{-3}$

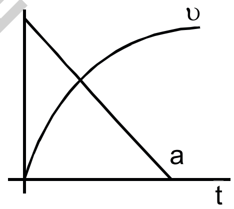
$[\sigma] = M^{-1}I^{2}L^{-3}T^{3}$

2. Which of the following option correctly describes the variation of the speed υ and acceleration 'a' of a point mass falling vertically in a viscous medium that applies a force $F = -k\upsilon$, where 'k' is a constant, on the body ? (Graphs are schematic and not drawn to scale)

(1)


(2*)


(3)


(4)


Sol. $F = -kv \Rightarrow F_{net} = mg - kv \Rightarrow ma = mg - kv$

$a = g - \frac{k}{m}v \Rightarrow \frac{k}{m} = C \Rightarrow a = g - Cv$

$\frac{da}{dt} = -Ca \Rightarrow \int_{a_0}^{a}\frac{da}{a} = -\int_{0}^{t}Cdt \Rightarrow \ell n\left(\frac{a}{a_0}\right) = -Ct$

$a = a_0e^{-Ct}$

$\frac{dv}{dt} = a = g - Cv$

$$\int_{v_0}^{v} \frac{dv}{g - Cv} = \int_0^t a\,dt$$

$$\left(\frac{1}{-C}\right)\ell n\frac{(g - Cv)}{g - Cv_0} = at$$

$$g - Cv = (g - Cv_0)\, e^{-Cat}$$

$$g - (gCV_0)e^{-Cat} = Cv$$

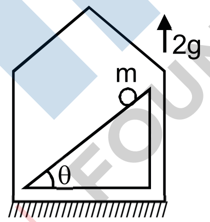
$$v = \frac{1}{C}(g - ge^{-Cat} + CV_0e^{-Cat})$$

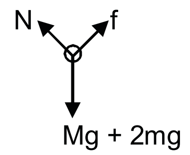
$$v = \frac{1}{C}\, g(1 - e^{-Cat}) + Cv_0e^{-Cat}$$

3. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is :

(1*) $\tan\theta$ (2) $2\tan\theta$ (3) $3\tan\theta$ (4) $\tan 2\theta$

Sol. in the frame of the rocket





$3mg\sin\theta - f = 0$

$3mg\cos\theta = N$

$f \geq \mu_{min} N$

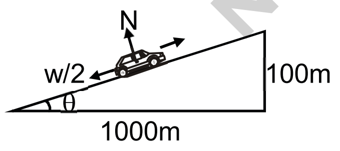
$3\,mg\sin\theta = \mu min\ 3\,mg\cos\theta$

$\mu_{min} = \tan\theta$

4. A car of weight W is on an inclined road that rises by 100 m over a distance of 1 km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of 10 ms^{-1}, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed υ then value of υ is:

(1) 20 ms^{-1} (2*) 15 ms^{-1} (3) 10 ms^{-1} (4) 5 ms^{-1}

Sol.

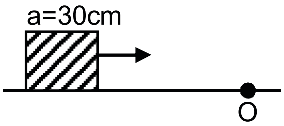


$$\text{Uphill } P = \vec{F}.\vec{v} = \left(\frac{W}{20} + W\sin\theta\right)v = \left(\frac{W}{20} + W0.099\right)v = \left(\frac{W}{20} + W0.099\right)10$$

$$\text{Downhill } \frac{P}{2} = \left(W \times 0.099 - \frac{W}{20}\right) v \Rightarrow 0.149 \times 5 = 0.049\, v$$

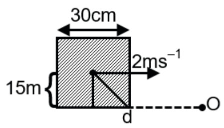
$v = 15\ ms^{-1}$

5. A cubical block of side 30 cm is moving with velocity 2 ms^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is :



(1*) 5.0 (2) 6.7 (3) 9.4 (4) 13.3

Sol.



$I_O = I_{cm} + mr^2$

$= \frac{m\ell^2}{12} + \frac{m\ell^2}{12} + mr^2$

$= \frac{m}{6}\frac{9}{100} + \frac{m}{2}\frac{9}{100} = m \times \frac{9}{100}\left(\frac{1}{6} + \frac{9}{6}\right) = m \times \frac{6}{100} = 0.06m$

Using conservation of Angular momentum about O

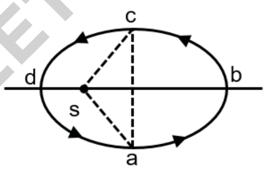
($\therefore$ angular impulse of mg is negligible)

$mvr = I\omega$

$m2 \times \frac{15}{100} = m \times 0.06 \times \omega$

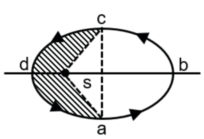
$\therefore\ \omega = 5\ rad\text{-}s^{-1}$

6. Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then :



(1) $t_1 = t_2$ (2) $t_1 = 2t_2$ (3*) $t_1 = 3t_2$ (4) $t_1 = 4t_2$

Sol.



$A_{cSa} = \frac{1}{4}A_{abcd}$

$A_{cSa} = \frac{1}{4} A_{abcd}$

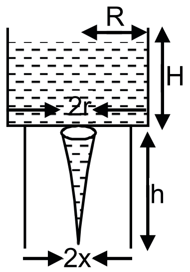
$A_{abcS} = A_{cSa} + A_{abc} = \left(\frac{1}{4} + \frac{1}{2}\right) A_{abcd}$

$A_{abcS} = \frac{3}{4} A_{abcd} \propto t_1$ (By Kepler's second law)

$A_{adaS} = \frac{1}{4} A_{abcd} \propto t_2$

$t_1 : t_2 = 3 : 1$

$\frac{t_1}{t_2} = \frac{3}{1} \Rightarrow t_1 = 3t_2$

7. 

Consider a water jar of radius R that has water filled up to height H and is kept on a stand of height h (see figure). Through a hole of radius r (r << R) at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross-section of water stream when it hits the ground is x. Then :

(1) $x = r\left(\frac{H}{H+h}\right)$ (2) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$ (3*) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$ (4) $x = r\left(\frac{H}{H+h}\right)^{2}$

Sol. Using equation of continuity

$\pi r^2 \sqrt{2gH} = \pi x^2 \sqrt{2g(H+h)}$ $x = r\left(\frac{H}{H+h}\right)^{1/2}$

$x = r\left(\frac{H}{H+h}\right)^{1/2}$

8. 200 g water is heated from 40ºC to 60ºC. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water=4184 J/kg/K) :

(1) 8.4 kJ (2) 4.2 kJ (3*) 16.7 kJ (4) 167.4 kJ

Sol. 200 g $40°C \rightarrow 60°C$

$\Delta U = mC_v\Delta T = 200 \times 10^{-3}$ kg × 418 J/kg/K

$\Delta U = 200 \times 4184 \times 20 \times 10^{-3}$ J

$\Delta U = 4 \times 4184$ J = 16.7 kJ

9. The ratio of work done by an ideal monoatomic gas to the heat supplied to it in an isobaric process is :

(1) $\frac{3}{5}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4*) $\frac{2}{5}$

Sol. Isobaric process

$Q = nCp\Delta T$

$W = P\Delta V$

$PV = nRT$

$P\Delta V = nR\Delta T$

$Q = nCP\,\frac{P\Delta V}{nR}$

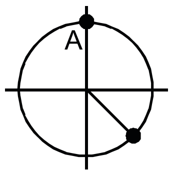
$Q = \frac{nCp}{nR}\,P\Delta V$

$$\frac{W}{Q} = \frac{1}{nCp/nR} = \frac{nR}{nCp} = \frac{R}{Cp} = \frac{Cp - Cv}{Cp} = 1 - \frac{Cv}{Cp} = \frac{2}{3}$$

10. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time t = 0 one particle has displacement A while the other one has displacement $\frac{-A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is :

(1*) $\frac{T}{6}$ (2) $\frac{5T}{6}$ (3) $\frac{T}{3}$ (4) $\frac{T}{4}$

Sol. $A\cos\omega t = +A\sin\left(\omega t - \frac{\pi}{6}\right)$



$\cos\omega t = +\left[\sin\omega t\frac{\sqrt{3}}{2} - \cos\omega t\frac{1}{2}\right]$

$\frac{3}{2}\cos\omega t = +\sin\omega t\,\frac{\sqrt{3}}{2}$

$\tan\omega t = +\sqrt{3}$

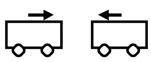
$\omega t = +\sqrt{3}$

$\omega t = \tan^{-1}(+\sqrt{3}) = \frac{\pi}{6}$

$$\frac{2\pi}{T} = \frac{\pi}{3} \Rightarrow t = \frac{T}{6}$$

11. Two engines pass each other moving in opposite directions with uniform speed of 30 m/s. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 m/sec :

(1) 450 Hz (2) 540 Hz (3*) 648 Hz (4) 270 Hz

Sol. 

$f = 540\text{ Hz}\left(\frac{v+v_0}{v-v_s}\right)$

$f = 540\left(\frac{330+30}{330-30}\right)$

f = 648 Hz

12. The potential (in volts) of a charge distribution is given by

$V(z) = 30 - 5z^2$ for $|z| \le 1$m

$V(z) = 35 - 10\,|z|$ for $|z| \ge$ m.

V(z) does not depend on x and y. If this potential is generated by a constant charge per unit volume ρ_0 ((in units of $\in_0$) which is spread over a certain region, then choose the correct statement.

(1*) $\rho_0 = 10\ \in_0$ for $|z| \le 1$ m and $\rho_0 = 0$ elsewhere

(2) $\rho_0 = 20\ \in_0$ in the entire region

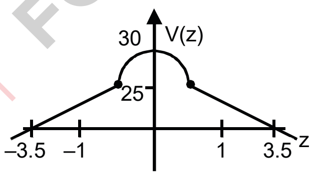
(3) $\rho_0 = 40\ \in_0$ in the entire region

(4) $r_0 = 20\ \in_0$ for $|z| \le 1$ m and $\rho_0 = 0$ elsewhere

Sol. $V(z) = 30 - 5z^2$ $\quad |z| \le 1$m

$V(z) = 35 - 10\,|z|$ $\quad |z| \ge$ m

$E = -\frac{\partial V}{\partial z}$



$E = +10z$ $\quad |z| \le 1$m

$E = 10$ $\quad z \ge 1$m

$E = -10$ $\quad z \le -1$m

$\vec{\nabla}\cdot\vec{E} = \frac{\rho_{enc}}{\varepsilon_0}$

for z > 1m or z < –1m

$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \frac{\rho_{enc}}{\varepsilon_0} = 0$

$\rho_{enc} = 0\ |z| \ge 1$m

$\vec{\nabla}\cdot\vec{E} = +10 = \frac{\rho_{enc}}{\varepsilon_0}$

$\rho_{enc} = 10\,\varepsilon_0\ |z| \le 1$m

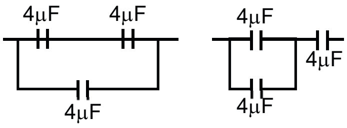
13. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by connecting them :

(1) all in series

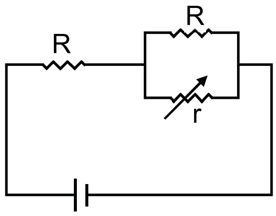
(2*) two in series and one in parallel

(3) all in parallel

(4) two in parallel and one in series

Sol. 

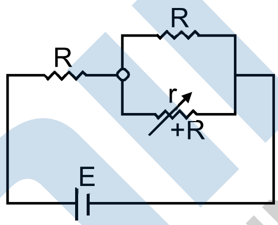
2 in series, one in parallel

14. 

In the circuit shown, the resistance r is a variable resistance. If for r = f R, the heat generation in r is maximum then the value of f is :

(1) $\frac{1}{4}$ (2*) $\frac{1}{2}$ (3) $\frac{3}{4}$ (4) 1

Sol. $R_{eq} = \frac{fRR}{R + fR} + R$



$R_{eq} = \frac{fR}{(1+f)} + R = \frac{(2f+1)R}{1+f}$

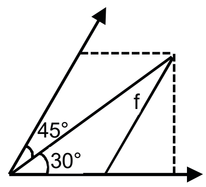
heat generation = $\frac{v_r^2 t}{r}$

$H = \frac{E^2 t}{R} \frac{f}{(2f+1)^2}$

15. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75º. One of the fields has a magnitude of 15 mT. The dipole attains stable equilibrium at an angle of 30º with this field. The magnitude of the other field (in mT) is close to :

(1*) 11 (2) 36 (3) 1 (4) 1060

Sol. $\tan 3\theta = \frac{f \sin 75^\circ}{f \cos 75^\circ + 15\text{mT}}$



f cos 75° tan 30° + 15 mT tan 30° = f sin 75°

f (sin 75° – cos 75° tan 30°) = 15mT tan 30°

$f = \frac{15\text{mT} \tan 30^\circ}{\sin 75^\circ - \cos 75^\circ \tan 30^\circ}$

$\Rightarrow f = \frac{15 \times \tan 30^\circ}{0.516} = 11$ mT

16. A 50 Ω resistance is connected to a battery of 5 V. A galvanometer of resistance 100 Ω is to be used as an ammeter to measure current through the resistance, for this a resistance r_s is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of the current without the ammeter in the circuit ?

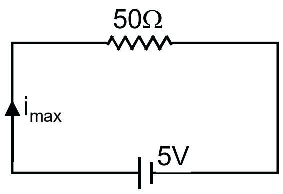
(1*) $r_s = 0.5\Omega$ in parallel with the galvanometer

(2) $r_s = 0.5\Omega$ in series with the galvanometer

(3) $r_s = 1\Omega$ in series with galvanometer

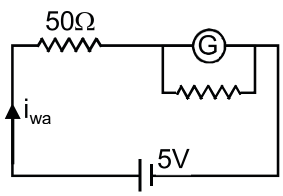
(4) $r_s = 1\Omega$ in parallel with galvanometer

Sol.



without ammeter $i_{max} = \frac{5}{50} = 0.1A$

A series connection not possible as that would alter the current a lot.



$$i_{wA} = \frac{5}{50 + \frac{r_s \times 100}{r_s + 100}} = \frac{r_{s+100}}{30r_s + 1000}$$

$$\frac{(5 - 50i_{wA})}{100} \le 0.01 \times 0.1$$

$5 - 50\ i_{wA} \le 0.1$

$$4.9 \le 50 \times \frac{r_s + 100}{30r_s + 1000}$$

$$\frac{4.9}{50} \le \frac{r_s + 100}{30r_s + 1000}$$

$$\frac{0.098}{5} \le \frac{r_s + 100}{30rs + 1000} \Rightarrow 30 \times 0.098\ rs + 98 \le rs + 100$$

$1.94\ r_s \le 2$

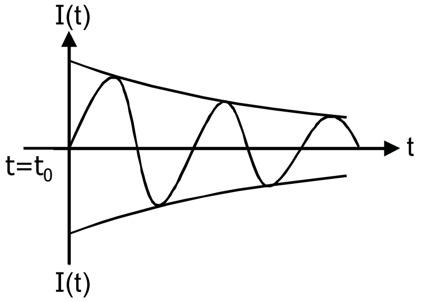
$r_s \le 2/1.94$

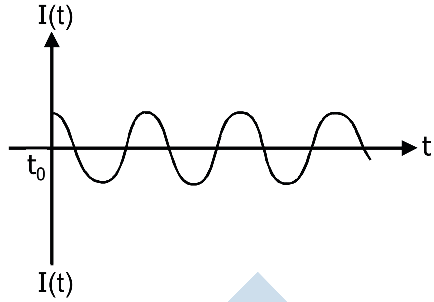
$r_s \le 1.2$

smaller r_s is better

$\therefore\ r_s = 0.5\Omega$

17. A series LR circuit is connected to a voltage source with $V(t)=V_0 \sin\Omega t$. After very large time, current I(t) behaves as $\left(t_0 >> \frac{L}{R}\right)$:

(1)


(2*)


(3)


(4)


Sol. $V(t) = V_0 \sin \Omega t$

After a long time, the transients settle down and the current oscillates with Ω frequency.

18. Microwave oven acts on the principle of :

(1) transferring electrons from lower to higher energy levels in water molecule

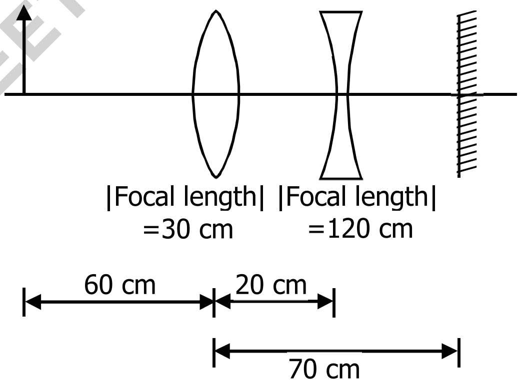
(2*) giving rotational energy to water molecules

(3) giving vibrational energy to water molecules

(4) giving translational energy to water molecules

Sol. Energy of microwaves lie in range of vibration energy of water molecules.

19. A convex lens, of focal length 30 cm, a concave lens of focal length 120 cm, and a plane mirror are arranged as shown. For an object kept at a distance of 60 cm from the convex lens, the final image, formed by the combination, is a real image, at a distance of :



(1*) 60 cm from the convex lens

(2) 60 cm from the concave lens

(3) 70 cm from the convex lens

(4) 70 cm from the concave lens

Sol. Applying the lens and mirror equations

20. In Young's double slit experiment, the distance between slits and the screen is 1.0 m and monochromatic light of 600 nm is being used. A person standing near the slits is looking at the fringe pattern. When the separation between the slits is varied, the interference pattern disappears for a particular distance d_0 between the slits. If the angular resolution of the eye is $\frac{1^\circ}{60}$, the value of d_0 is close to :

(1) 1 mm (2*) 2 mm (3) 4 mm (4) 3 mm

Sol. Angular fringe width $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

$$\frac{\lambda}{d_0} = \frac{1^\circ}{60} = \frac{\pi}{180 \times 60}$$

$$d_0 = \lambda\left(\frac{180 \times 60}{\pi}\right)$$

$= 2 \times 10^{-3}$ m = 2mm.

21. When photons of wavelength λ_1 are incident on an isolated sphere, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice that of the above value. If light of wavelength λ_3 is used then find the stopping potential for this case :

(1) $\frac{hc}{e}\left[\frac{1}{\lambda_3} - \frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$ (2) $\frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$

(3*) $\frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{1}{2\lambda_1}\right]$ (4) $\frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{1}{\lambda_1}\right]$

Sol. $KE_{max} = \frac{hc}{\lambda} - \phi$

$eV = \frac{hc}{\lambda_1} - \phi$

$eV = \frac{hc}{\lambda_2} - \phi$

$eV'' = \frac{hc}{\lambda_3} - \phi$

$$\frac{hc}{\lambda_2} - \phi = \left(\frac{hc}{\lambda_1} - \phi\right)$$

$$\frac{hc}{\lambda_2} - \phi = \frac{3hc}{\lambda_1} - 3\phi \Rightarrow 2\phi = \frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$\phi = \frac{3}{2}\frac{hc}{\lambda_1} - \frac{hc}{2\lambda_2}$$

$$eV'' = \frac{hc}{\lambda_3} - \frac{3}{2}\frac{hc}{\lambda_1} + \frac{hc}{2\lambda_2}$$

$$V'' = \left[\frac{1}{\lambda_3} - \frac{3}{2\lambda_1} + \frac{1}{2\lambda_2}\right]$$

22. A hydrogen atom makes a transition from n = 2 to n = 1 and emits a photon. This photon strikes a doubly ionized lithium atom (z = 3) in excited state and completely removes the orbiting electron. The least quantum number for the excited state of the ion for the process is :

(1) 2 (2) 3 (3*) 4 (4) 5

Sol. $E = \frac{-13.6Z^2 eV}{h^2}$

$$\Delta E = -13.6\left[\frac{1}{2^2} - \frac{1}{1^2}\right]eV$$

$$= +13.6\left[\frac{3}{4}\right]eV$$

$= 3.4 \times 3$ eV

$\Delta E = 10.2$ eV

$$E_{Li,\,3} = -\frac{13.6 \times 3^2}{n^2} = \frac{-13.6 \times 9}{n^2}$$

$E_{Li,3} + \Delta E \geq 0$

$$10.2 \text{ eV} \geq \frac{13.6 \times 9}{n^2} \text{ eV}$$

$$n^2 \geq \frac{13.6 \times 9}{10.2}$$

$n^2 \geq 4$

23. The truth table given in figure represents :

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(1) AND – Gate (2*) OR – Gate (3) NAND – Gate (4) NOR – Gate

Sol. from truth table its clear.

24. An audio signal consists of two distinct sounds : one a human speech signal in the frequency band of 200 Hz to 2700 Hz, while the other is a high frequency music signal in the frequency band of 10200 Hz to 15200 Hz. The ratio of the AM signal bandwidth required to send both the signals together to the AM signal bandwidth required to send just the human speech is :

(1) 3 (2) 5 (3*) 6 (4) 2

Sol. Band width for both signals 15200 Hz – 200 Hz = 15000 Hz

Band width for human speech 2700 Hz – 200 Hz = 2500 Hz

The ratio $= \frac{15000}{2500} = 6$

25. A simple pendulum made of a bob of mass m and a metallic wire of negligible mass has time period 2 s at T = 0ºC. If the temperature of the wire is increased and the corresponding change in its time period is plotted against its temperature, the resulting graph is a line of slope S. If the coefficient of linear expansion of metal is áthen the value of S is :

(1) α (2*) $\frac{\alpha}{2}$ (3) 2α (4) $\frac{1}{\alpha}$

Sol. $t_0 = 2\pi\sqrt{\frac{\ell_0}{g}}$

$\ell = \ell_0 (1 + \alpha\Delta T)$

$t = 2\pi\sqrt{\frac{\ell_0(1+\alpha\Delta T)}{g}}$

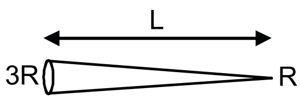
$t = 2\pi \sqrt{\frac{\ell_0}{g}}\left(1+\frac{\alpha}{2}\Delta T\right)$

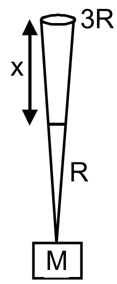
$S = \frac{\alpha}{2}$

26. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unexpended length L. The radii, at the upper and lower ends of this conical wire, have values R and 3R, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal :

(1) $L\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$ (2*) $L\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$ (3) $L\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$ (4) $L\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

Sol. $r = 3R - \frac{x}{L}2R \rightarrow$ radius at x





$\frac{Mg/\pi r^2}{\Delta x/dx} = Y$

$\frac{\Delta x}{dx} = \frac{Mg}{Y\pi r^2}$

$\Delta x = \frac{Mg}{4\pi\left(3R - \frac{x2R}{L}\right)^2}dx$

$\int_0^{\Delta L}\Delta x = \int_0^L \frac{Mg}{\pi(3R - xc)^2}dx$ $c = \frac{2R}{L}$

$\Delta L = \frac{Mg}{Y\pi}\int_0^L (3R - xc)^{-2}dx = \frac{Mg}{Y\pi c}(3R-xc)^{-1}\Big|_0^L = \frac{Mg}{Y\pi c}\left[\frac{1}{(3R-2R)} - \frac{1}{3R}\right]$

$\Delta L = \frac{MgL}{Y\pi 2R}\left[\frac{3-1}{3R}\right] \Rightarrow \frac{ML}{3Y\pi R^2}$

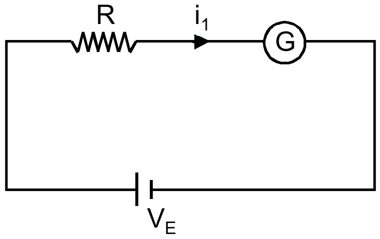
$\therefore$ Equilibrium length = L + ΔL = L + $\frac{LMg}{3\pi YR^2} = L\left(1+\frac{Mg}{3\pi YR^2}\right)$

27. To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection then G, R and S are related by the equation :

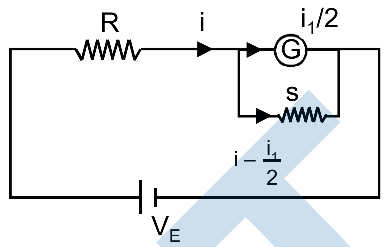
(1) 2S (R + G) = RG (2*) S (R + G) = RG (3) 2S = G (4) 2G = S

Sol.

Case–1



Case–2



$$i_1 = \frac{V_E}{R+G}$$

$$i_1 = \frac{V_E}{R+\frac{GS}{G+S}}$$

$$\frac{i_1}{2}G = \left(i-\frac{i_1}{2}\right)S$$

i_1(G + S) = 2 i S

substituting i_1 and i we get

S(R+G) = RG

28. To find the focal length of a convex mirror, a student records the following data :

Object Pin	Convex Lens	Convex Mirror	Image Pin
22.2cm	32.2cm	45.8cm	71.2cm

The focal length of the convex lens is f_1 and that of mirror is f_2. Then taking index correction to be negligibly small, f_1 and f_2 are close to :

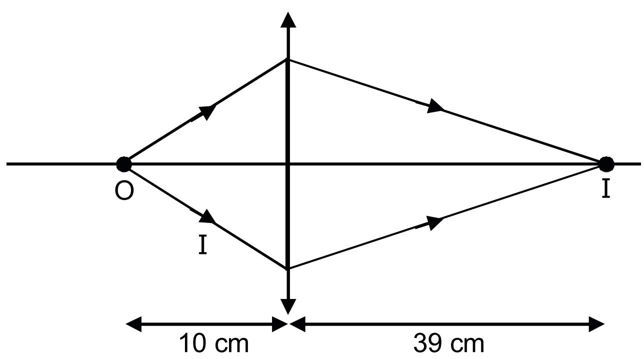
(1) f_1 = 12.7 cm f_2 = 7.8 cm

(2*) f_1 = 7.8 cm f_2 = 12.7 cm

(3) f_1 = 7.8 cm f_2 = 25.4 cm

(4) f_1 = 15.6 cm f_2 = 25.4 cm

Sol. For convex lens



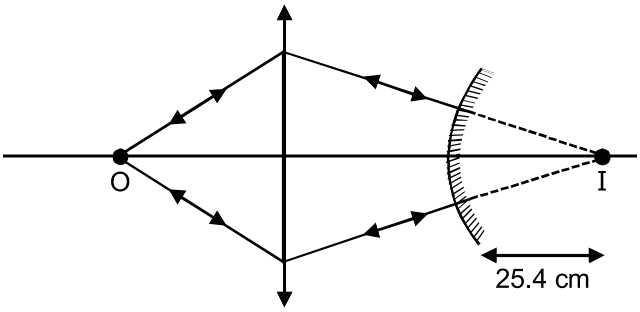
u = –10 cm

v = 39 cm

$f_1 = \frac{uv}{u-v}$

$= \frac{390}{49} = 7.8$ cm

For convex mirror



R = 25.4 cm

f_2 = 12.7 cm

29. An experiment is performed to determine the I - V characteristics of a Zener diode, which has a protective resistance of R=100 Ω, and a maximum power of dissipation rating of 1 W. The minimum voltage range of the DC source in the circuit is :

(1) 0 – 5 V (2) 0 – 8 V (3) 0 – 12 V (4*) 0 – 24 V

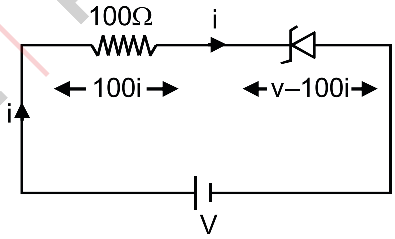
Ans. $P_{zener} = (v-100i)I = 1$

$Vi - 100i^2 = 1$

$100i^2 - vi + 1 = 0$

$v^2 - 4(100) \geq 0$

$v \geq 20$



30. An unknown transistor needs to be identified as a npn or pnp type. A multimeter, with +ve and –ve terminals, is used to measure resistance between different terminals of transistor. If terminal 2 is the base of the transistor then which of the following is correct for a pnp transistor ?

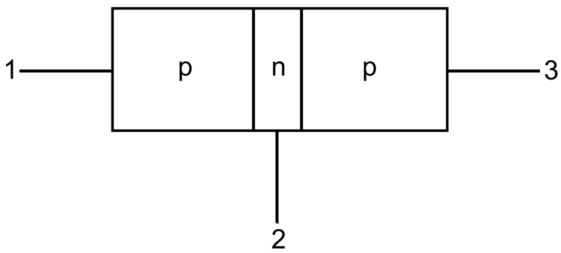
(1) +ve terminal 1, –ve terminal 2, resistance high

(2*) +ve terminal 2, -ve terminal 1, resistance high

(3) +ve terminal 3, -ve terminal 2, resistance high

(4) +ve terminal 2, -ve terminal 3, resistance low

Sol.



when pn junction is forward biased resistance is law.When pn junction reverse biased resistance is high

PART-B-CHEMISTRY

31. The amount of arsenic pentasulphide that can be obtained when 35.5 g arsenic acid is treated with excess H_2S in the presence of conc. HCl (assuming 100 % conversion) is :

(1) 0.50 mol (2) 0.25 mol (3*) 0.125 mol (4) 0.333 mol

Sol. $H_3AsO_4 \xrightarrow{H_2s/HCl} As_2S_5$

A

32. At very high pressure, the compressibility factor of one mole of a gas is given by:

(1) $\frac{pb}{RT}$ (2*) $1+\frac{pb}{RT}$ (3) $1-\frac{pb}{RT}$ (4) $1-\frac{b}{(VRT)}$

Sol. According to Vander waal's equation for one mole of gas

$$\left(P+\frac{a}{v^2}\right)(V-b)=RT\,b$$

at high pressure $\frac{a}{V^2}$ can be neglected with respect to P,

$$\therefore \quad P+\frac{a}{V^2} \approx P$$

P(V – b) = RT

PV – Pb = RT

PV = RT + Pb

divided on RT on both side,

$$Z=1+\frac{Pb}{RT}$$

33. The total number of orbitals associated with the principal quantum number 5 is:

(1) 5 (2) 10 (3) 20 (4*) 25

Sol. n = 5

Possible subshell are

$\Rightarrow$ 5s, 5p, 5d, 5f, 5g

$\therefore$ Total number of orbital = 1 + 3 + 5 + 7 + 9 = 25

34. Which intermolecular force is most responsible in allowing xenon gas to liquefy?

(1) Dipole - dipole (2) Ion - dipole

(3*) Instantaneous dipole - induced dipole (4) Ionic

Sol. Instantaneous dipole-induced dipole forces are most responsible in allowing xenon gas to liquidfy.

35. A reaction at 1 bar is non-spontaneous at low temperature but becomes spontaneous at high temperature. Identify the correct statement about the reaction among the following:

(1) Both ΔH and ΔS are negative (2*) Both ΔH and ΔS are positive

(3) ΔH is positive while ΔS is negative (4) ΔH is negative while ΔS is positive

Sol. $\Delta G = \Delta H - T.\Delta S$

If ΔH & ΔS are both positive, then ΔG may be negative at high temperature hence reaction becomes spontaneous at high temperature

36. The solubility of N_2 in water at 300 K and 500 torr partial pressure is 0.01 g L^{-1}. The solubility (in g L^{-1}) at 750 torr partial pressure is :

(1) 0.0075 (2*) 0.015 (3) 0.02 (4) 0.005

Sol. According to hennery law

$\frac{P_1}{P_2} = \frac{S_1}{S_2}$ $\therefore$ S_1 & S_2 are solubility of gas (g/L)

$\frac{500}{750} = \frac{0.01}{S_2}$

$\therefore S_2 = \frac{750 \times 0.01}{500} = 0.015 \text{ g/L}$

37. For the reaction,

A(g) + B(g) $\longrightarrow$ C(g) + D(g), $\Delta H°$ and $\Delta S°$ are respectively, –29.8 kJ mol^{-1} and –0.100 kJ K^{-1} mol^{-1} at 298 K. The equilibrium constant for the reaction at 298 K is:

(1) 1.0×10^{-10} (2) 1.0×10^{10} (3) 10 (4*) 1

Sol. $\Delta G° = \Delta H° - T.\Delta S°$
$= -29.8 + 298 \times (0.1)$
$= -29.8 + 29.8$
$\because \Delta G° = 0$
apply relation between $\Delta G°$ & K_{eq}
$\Delta G° = -RT \ln K_{eq}$
$\therefore K_{eq} = 1$

38. What will occur if a block of copper metal is dropped into a beaker containing a solution of 1M $ZnSO_4$?

(1) The copper metal will dissolve and zinc metal will be deposited

(2) The copper metal will dissolve with evolution of hydrogen gas.

(3) The copper metal will dissolve with evolution of oxygen gas.

(4*) No reaction will occur.

Sol. if a block of copper metal is dropped into a beaker containing solution of 1 M $ZnsSO_4$, no reaction will occur because $E°_{Zn^{2+}/Zn} = -0.76\text{ V}$

$E°_{Cu^{2+}/Cu} = +0.34\text{ V}$

39. The reaction of ozone with oxygen atoms in the presence of chlorine atoms can occur by a two step process shown below:

$O_3(g) + Cl^•(g) \longrightarrow O_2(g) + ClO^•(g)$ ...... (i) $k_i = 5.2 \times 10^9$ L mol^{-1} s^{-1}

$ClO^•(g) + O^•(g) \longrightarrow O_2(g) + Cl^•(g)$ ......(ii) $k_{ii} = 2.6 \times 10^{10}$ L mol^{-1} s^{-1}

The closest rate constant for the overall reaction $O_3(g) + O^•(g) \longrightarrow 2O_2(g)$ is:

(1*) 5.2×10^9 L mol^{-1} s^{-1} (2) 2.6×10^{10} L mol^{-1} s^{-1}

(3) 3.1×10^{10} L mol^{-1} s^{-1} (4) 1.4×10^{20} L mol^{-1} s^{-1}

Sol. The rate constant of overall reaction depends slowest step. Hence equation(i) is slowest step. Option(2) is correct.

40. A particular adsorption process has the following characteristics : (i) It arises due to van der Waals forces and (ii) it is reversible. Identify the correct statement that describes the above adsorption process:

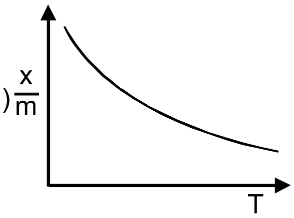
(1) Enthalpy of adsorption is greater than 100 kJ mol^{-1}.

(2*) Energy of activation is low.

(3) Adsorption is monolayer.

(4) Adsorption increases with increase in temperature

Sol. Adsorption aries due to Vander waal forces & reversible, hence it should be physisorption (physical adsorption).
(i) Enthalpy of physisorption is low (20 – 40 kJ/mol)
(ii) In physisorption multimolecular layer form.

(iii) 

Physisorption decreases with increase in temperature.

(iv) Physisorption required number activation energy.
Hence answer is (4)

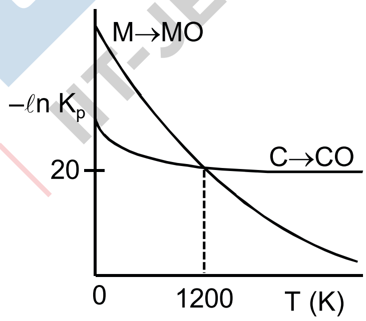
41. The non-metal that does not exhibit positive oxidation state is:

(1) Oxygen (2) Iodine (3) Chlorine (4*) Fluorine

Sol. Fluorine is the most electronegative element in periodic table hence it shows –1 oxidation state in all its compounds.

42. The plot shows the variation of –ln K_P versus temperature for the two reactions.

$$M(s) + \frac{1}{2}\,O_2\,(g) \longrightarrow MO\,(s) \text{ and } C(s) + \frac{1}{2}\,O_2\,(g) \longrightarrow CO(s)$$



Identify the correct statement:

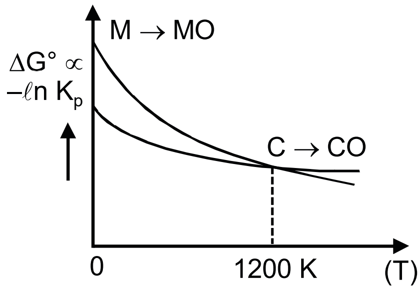
(1) At T > 1200 K, carbon will reduce MO (s) to M(s).

(2) At T < 1200 K, the reaction MO (s) + C(s) $\longrightarrow$ M(s) + CO (g) is spontaneous

(3) At T < 1200 K, oxidation of carbon is unfavourable

(4*) Oxidation of carbon is favourable at all temperatures.

Sol. According to Ellingham diagram, as given



At T < 1200, carbon will reduce $MO_{(s)}$ to $M_{(s)}$ hence, chemical reaction

$C_{(s)} + MO_{(s)} \longrightarrow M_{(s)} + CO_{(g)}$ is spontaneous.

43. Identify the incorrect statement regarding heavy water:

(1) It reacts with Al_4C_3 to produce CD_4 and $Al(OD)_3$.

(2*) It is used as a coolant in nuclear reactors

(3) It reacts with CaC_2 to produce C_2D_2 and $Ca(OD)_2$

(4) It reacts with SO_3 to form deuterated sulphuric acid (D_2SO_4)

Sol. Heavy water (D_2O) acts as moderator used to slow down the speed of neutrons in nuclear reactor, hence option (2) is incorrect.

44. The correct order of the solubility of alkaline-earth metal sulphates in water is:

(1) Mg < Ca < Sr < Ba
(2) Mg < Sr < Ca < Ba
(3) Mg > Sr > Ca > Ba
(4*) Mg > Ca > Sr > Ba

Sol. Solubility of sulphates of alkaline earth metal decreases down the group. Hence correct order of solubility is Mg > Ca > Sr > Ba

45. Match the items in column I with its main use listed in column II :

Column I	Column II
(A) Silica gel	(i) Transistor
(B) Silicon	(ii) Ion-exchanger
(C) Silicone	(iii) drying agent
(D) Silicate	(iv) Sealant

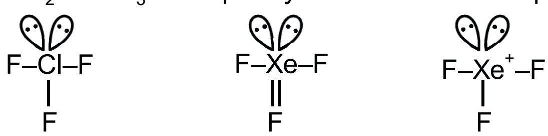
(1*) (A)-(iii), (B)-(i), (C)-(iv), (D)-(ii)
(2) (A)-(iv), (B)-(i), (C)-(ii), (D)-(iii)
(3) (A)-(ii), (B)-(iv), (C)-(i), (D)-(iii)
(4) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)

Sol. Based on theoretical fact.

46. The group of molecules having identical shape is:

(1) SF_4, XeF_4, CCl_4
(2*) ClF_3, $XeOF_2$, XeF_3^+
(3) BF_3, PCl_3, XeO_3
(4) PCl_5, IF_5, XeO_2F_2

Sol. ClF_3, $XeOF_2$ & XeF_3^+ are sp^3d hybridized with 2 lone pair e's, hence all have (T-shape) identical shape.



47. Which one of the following species is stable in aqueous solution?

(1) Cr^{2+}
(2) Cu^+
(3) MnO_4^{3-}
(4*) MnO_4^{2-}

Sol. (1) MnO_4^{2-} disproportionates in neutral or acidic solution.

$3MnO_4^{2-} + 4H^+ \longrightarrow 2\ MnO_4^{2-} + MnO_2 + 2H_2O$

(3) Many Cu^+ compounds are unstable in aqueous solution and undergo disproportionation as follows

$2Cu^+ \longrightarrow Cu^{2+} + Cu$

48. Which one of the following complexes will consume more equivalents of aqueous solution of $Ag(NO_3)$?

(1) $Na_3[CrCl_6]$ (2) $[Cr(H_2O)_5Cl]Cl_2$ (3*) $[Cr(H_2O)_6]Cl_3$ (4) $Na_2[CrCl_5(H_2O)]$

Sol. Complex $[Cr(H_2O)_6]Cl_3$ will consume more equivalents of aqueous solution of $Ag(NO_3)$.

49. Identify the correct trend given below:

[Atomic number : Ti = 22, Cr = 24 and Mo = 42]

(1) Δo of $[Cr(H_2O)_6]^{2+} > [Mo(H_2O)_6]^{2+}$ and Δo of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$

(2) Δo of $[Cr(H_2O)_6]^{2+} > [Mo(H_2O)_6]^{2+}$ and Δo of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$

(3*) Δo of $[Cr(H_2O)_6]^{2+} < [Mo(H_2O)_6]^{2+}$ and Δo of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$

(4) Δo of $[Cr(H_2O)_6]^{2+} < [Mo(H_2O)_6]^{2+}$ and Δo of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$

Sol. $\Delta_o \propto$ CFSE (Crystal field stabilization energy)

Δ_o of $[Cr(H_2O)_6]^{2+} < \Delta_o$ of $[Mo(H_2O)_6]^{2+}$

Because here Δ_o depends on Z_{eff} & Z_{eff} of 4d series is more than 3d series.

But Δ_o of $[Ti(H_2O)_6]^{3+} < \Delta_o$ of $[Ti(H_2O)_6]^{2+}$

50. BOD stands for :

(1) Biological Oxygen Demand (2) Bacterial Oxidation Demand

(3*) Biochemical Oxygen Demand (4) Biochemical Oxidation Demand

Sol. BOD stands for Biochemical oxygen demand.

51. An organic compound contains C, H and S. The minimum molecular weight of the compound containing 8% Sulphur is: [Atomic weight of S = 32 amu]

(1) 200 g mol^{-1} (2*) 400 g mol^{-1} (3) 600 g mol^{-1} (4) 300 g mol^{-1}

Sol. 8 g sulphur present in = 100 g of organic compound.

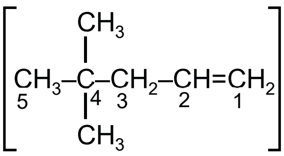
$\therefore$ 32 g sulphur present in = $\frac{100}{8} \times 32$ = 400 g of organic compound.

Hence, minimum molecular weight of compound = 400 g/mol

52. The hydrocarbon with seven carbon atoms containing a neopentyl and a vinyl group is:

(1) 2, 2-dimethyl-4-pentane (2) Isopropyl-2-butene

(3*) 4, 4-dimethylpentene (4) 2, 2-dimethyl-3-pentene

Sol. 

has seven carbon atoms containing a neopentyl and a vinyl group.

53. 5 L of an alkane requires 25 L of oxygen for its complete combustion. If all volumes are measured at constant temperature and pressure, the alkane is:

(1) Ethane (2*) Propane (3) Butane (4) Isobutane

Sol. $C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \longrightarrow nCO_2 + (n+1)H_2O$

5L 25 L

Since volumes are measured at constant T & P

So, Volume $\propto$ mole

$\therefore \quad n_{alkane} = \left(\frac{2}{3n+1}\right) \times n_{O_2}$

$5 = \frac{2}{3n+1} \times 25$

$\therefore \quad n = 3$

$\therefore \quad$ Alkane is propane (C_3H_8).

54. The gas evolved on heating CH_3MgBr in methanol is:

(1) HBr (2*) Methane (3) Ethane (4) Propane

Sol. $CH_3MgBr + CH_3–OH \longrightarrow (CH_3O)MgBr + CH_4\uparrow$ gas.

55. Bouveault -Blanc reduction reaction involves:

(1) Reduction of an acyl halide with H_2 / Pd.

(2*) Reduction of an ester with Na / C_2H_5OH

(3) Reduction of a carbonyl compound with Na / Hg and HCl

(4) Reduction of an anhydride with $LiAlH_4$

Sol. Reduction using Na in ethylalcohol is called Bouveault-Blanc reduction.

56. The test to distinguish primary, secondary and tertiary amines is:

(1) Carbylamines reaction (2*) $C_6H_5SO_2Cl$

(3) Sandmeyer's reaction (4) Mustard oil test

Sol. Benzene sulphonyl chloride (C6H5SO2Cl) is used to distinguish primary, secondary and tertiary amine.

57. **Assertion :** Rayon is a semisynthetic polymer whose properties are better than natural cotton.

Reason : Mechanical and aesthetic properties of cellulose can be improved by acetylation.

(1*) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.

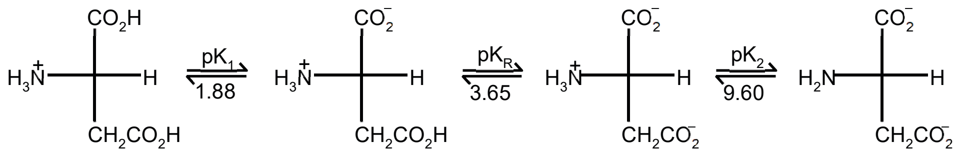
(2) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.

(3) Assertion is incorrect statement, but the reason is correct.

(4) Both assrtion and reason are incorrect.

Sol. Rayon is prepared by acetylation of cellulose.

58. Consider the following sequence for aspartic acid:



The pI (isoelectric point) of aspartic acid is :

(1) 1.88 (2) 3.65 (3) 5.74 (4*) 2.77

Sol. In given reaction sequence

$$PI = \frac{pK_1 + pK_R}{2}$$

$$= \frac{1.88 + 3.65}{2} = 2.77$$

59. The artificial sweetener that has the highest sweetness value in comparison to cane sugar is:

(1) Aspartame (2) Saccharin (3) Sucralose (4*) Alitame

Sol. Alitame is 2000 times sweeter than sucrose.

60. The most appropriate method of making egg-albumin sol is:

(1*) Break an egg carefully and transfer the transparent part of the content to 100 mL of 5 % w/V saline solution and stir well.

(2) Break an egg carefully and transfer only the yellow part of the content to 100 mL of 5 % w/V saline solution and stir well.

(3) Keep the egg in boiling water for 10 minutes. After removing the shell, transfer the white part of the content to 100 mL of 5 % w / V saline solution and homogenize with a mechanical shaker.

(4) Keep the egg in boiling water for 10 minutes. After removing the shell, transfer the yellow part of the content to 100 mL of 5 % w / V saline solution and homogenize with a mechanical shaker.

Sol. Only the transparent part of egg has albumin.

PART-C-MATHEMATICS

61. For $x \in R$, $x \neq 0$, $x \neq 1$, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, n = 0, 1, 2, ... Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is qual to :

(1) $\frac{8}{3}$ (2*) $\frac{5}{3}$ (3) $\frac{4}{3}$ (4) $\frac{1}{3}$

Sol. $f_0(x) = \frac{1}{1-x}$

$f_1(x) = f_0(f_0(x)) = \frac{1}{1-f_0(x)}; f_0(x) \neq 1$

$= \frac{1}{1-\frac{1}{1-x}}$ $x \neq 0$

$= \frac{1-x}{-x}$

$f_2(x) = f_0(f_1(x)) = \frac{1}{1-f_1(x)}; f_1(x) \neq 1$

$= \frac{1}{1+\frac{1-x}{x}} = x$

Similarly $f_3(x) = f_0(x)$

$f_4(x) = f_1(x)$

$f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = f_1(3) + f_1\left(\frac{2}{3}\right) + \frac{3}{2}$

$= 1 - \frac{1}{3} + 1 - \frac{3}{2} + \frac{3}{2} = \frac{5}{3}$

62. The point represented by 2 + i in the Argand plane moves 1unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by :

(1) 2 + 2i (2*) 1 + i (3) –1 – i (4) –2 – 2i

Sol. Let P(2 + i)

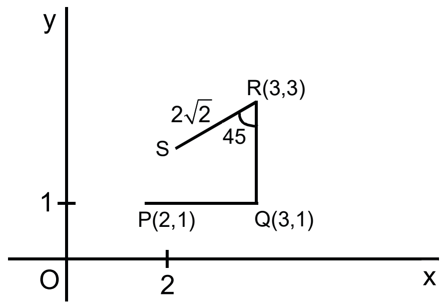
By rotation theorem

$$\frac{z-(3+3i)}{3+i-(3+3i)} = \frac{2\sqrt{2}}{2} e^{(-\pi/4)i}$$

$$\frac{z-3-3i}{-2i} = 1-i$$

Z – 3 – 3i = – 2i – 2

Z = 1 + i



63. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from –1, then |b| is equal to :

(1) $\sqrt{2}$ (2) 2 (3) 3 (4*) $\sqrt{3}$

Sol. $x^2 + bx . 1 = 0$ & $x^2 + x + b = 0$ have common root α.

$\Rightarrow \quad \alpha^2 + b\alpha - 1 = 0$

$\alpha^2 + \alpha + b = 0$

$\Rightarrow \quad \frac{\alpha^2}{b^2+1} = \frac{\alpha}{-(b+1)} = \frac{1}{(1-6)} \quad \Rightarrow \quad (b+1)^2 = (b^2+1)(1-b)$

$\Rightarrow \quad b^2 + 2b + 1 = b^2 - b^3 + 1 - b \quad \Rightarrow \quad b^3 + 3b = 0$

$\Rightarrow \quad b = 0$ or $b^2 = -3$

when b = 0 then common roots is (–1) hence b = 0 rejected.

So $b^2 = -3 \Rightarrow b = \pm\sqrt{3}i \quad \Rightarrow \quad |b| = \sqrt{3}$

64. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is

(1) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$ (3) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$ (4*) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

Sol. $P P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P^T P$

New $P^T Q^{2015} P = P^T \underbrace{PAP^T \; PAP^T \; ....PAP^T}_{2015 \text{ times}} P$

Because $= A^{2015}$

Now $A^2 - 2A + I = 0$

$\Rightarrow \quad A^n = nA - (n-1) I \quad \Rightarrow \quad A^{2015} = 2015 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - (2014) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

65. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is :

(1) 4 (2) 3 (3*) 2 (4) 1

Sol. $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

$\Rightarrow \cos^3x + \sin^3x + \sin^3x - 3\sin^2x\cos x = 0$

$\Rightarrow (\cos x + \sin x + \sin x)(\cos^2x + \sin^2x + \sin^2x - \cos x\sin x - \cos x\sin x - \sin^2x) = 0$

$\Rightarrow \cos x = -2\sin x$ or $\cos x = \sin x$

$\tan = -\frac{1}{2}$ $\tan = 1 \Rightarrow x = \pi/4$

$x = -\tan^{-1}\frac{1}{2}$ $\therefore$ two solutions

66. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is :

(1) $\frac{11!}{(2!)^3}$ (2) 110 (3) 56 (4*) 59

Sol. There are 1M, 3E, 1D, II, IT, 2R, 2A, 2N

R– –E– – – – – – – – – – –

rest of 11 letters can be arranged in $\frac{11!}{(2\,!)^3}$

67. For $x \in R$, $x \neq -1$, if $(1 + x)^{2016} + x(1 + x)^{2015} + x^2(1 + x)^{2014} + \ldots\ldots + x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to

(1*) $\frac{2017!}{17!\ 2000!}$ (2) $\frac{2016!}{17!\ 1999!}$ (3) $\frac{2017!}{2000!}$ (4) $\frac{2016!}{16!}$

Sol. $\sum_{i=0}^{2016} c_i.x^i = (1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots\ldots + x^{2016}$

$$= \frac{(1+x)^{2016}\left(1-\left(\frac{x}{1+x}\right)^{2017}\right)}{1-\frac{x}{1+x}}$$

$$= \frac{\frac{(1+x)^{2016}}{1} - \frac{x^{2017}}{(1+x)}}{\frac{x+1-x}{1+x}} = \frac{(1+x)^{2017} - x^{2017}}{1}$$

$\therefore a_{17} = {}^{2017}c_{17} = \frac{2007!}{17!\ 2000!}$ s

68. Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to :

(1) 270 (2) 258 (3) 342 (4*) 216

Sol. $x + y + z = 12$

$x^3y^4z^5 = (0.1)(600)^{3]}$

$$\frac{3\left(\frac{x}{3}\right)+4\left(\frac{y}{4}\right)+5\left(\frac{z}{5}\right)}{12} \geq \left\{\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)\left(\frac{z}{5}\right)^5\right\}^{1/12}$$

$$1 \geq \frac{x^3y^4z^5}{(60)^3(4\times25)}$$

$x^3y^4\,z^5 \leq (0.1)\,(600)^3$
But $x^3y^4\,z^5 = (10.1)\,(600)^3$
Clearly AM = GM

Hence $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ $\Rightarrow x = 3,\ y = 4,\ z = 5$

$\Rightarrow x^3 + y^3 + z^3 = 27 + 64 + 125 = 216$

69. The value of $\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$ is equal to

(1) 560 (2*) 680 (3) 1240 (4) 1085

Sol. $\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)=\sum_{r=1}^{15} r^2\left(\frac{15-r+1}{r}\right)=\sum_{r=1}^{15} r(16-r)=16\left(\frac{15\times16}{2}\right)-\frac{15\times16\times31}{6}=\frac{15\times16}{6}(17)=680.$

70. If $\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, then 'a' is equal to

(1) 2 (2*) $\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

Sol. $L=\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}$ must be of the form 1^∞

$$L=e^{\lim_{x\to\infty}\left(\frac{4}{x}-\frac{4}{x^2}\right)2x}$$

$$\Rightarrow \quad L=e^{\lim_{x\to\infty}\frac{2(ax-4)}{4}}$$

$= e^{2a} = e^3$

$a=\frac{3}{2}$

71. If the function $f(x)=\begin{cases}-x, & x<1\\ a+\cos^{-1}(x+b) & 1\leq x\leq 2\end{cases}$ is differentiable at x = 1, then $\frac{a}{b}$ is equal to :

(1) $\frac{\pi-2}{2}$ (2) $\frac{-\pi-2}{2}$ (3*) $\frac{\pi+2}{2}$ (4) $-1-\cos^{-1}(2)$

Sol. L.H.L. at x = 1 is – 1
R.H.L at x = 1 is $a + \cos^{-1}(1 + b)$
$\Rightarrow -1 = a + \cos^{-1}(1 + b)$
$\text{Cos}^{-1}(1 + b) = -1 - a$ ....(i)
now L.H.D. at x = 1 is –1

R.H.D at x = 1 is $\frac{-1}{\sqrt{1-(1+b)^2}}$

$\Rightarrow (1 + b)^2 = 0 \Rightarrow b = -1$

Now $\cos^{-1}(1 . 1) = -1 - a$

$a = -1 - \frac{\pi}{2}$

$\frac{a}{b} = \frac{-(2+\pi)}{2(-1)} = \frac{2+\pi}{2}$

72. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$, $t \in R$, meets the curve again at a point Q, then the coordinates of Q are :

(1*) $(t^2 + 3, -t^3 - 1)$ (2) $(4t^2 + 3, -8t^3 - 1)$ (3) $(t^2 + 3, t^3 - 1)$ (4) $(16t^2 + 3, -64t^3 - 1)$

Sol. $P(x = 4t^2 + 3, y = 8t^2 - 1)$

let $Q(4t_1^2 + 3, 8y_1^3 - 1)$

at P, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{24t^2}{8t} = 3t$

$\therefore$ tangent at P is $y - 8t^3 + 1 = 3t (x . 4t2 . 3)$

Q will satisfy it

$\therefore \quad 8t_1^3 - 8t^3 = 3t (4t_1^2 - 4t^2)$

$8(t_1 - t)(t_1^2 + t_1t + t^2) = 3t. 4(t_1 - t)(t_1 + t)$

$2(t_1^2 + t_1t + t^2) = 3t(t_1 + t)$

$2t_1^2 + 2t_1t + 2t^2 = 3t\, t_1 + 3t^2$

$2t_1^2 - t_1t - t^2 = 0$

$(t_1 - t)(2t_1 + t) = 0$

$t_1 = -\frac{1}{2}$

$\therefore \quad Q(t^2 + 3, -t^3 - 1)$ Ans. (1)

73. The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is :

(1) $\frac{\sqrt{19}}{2}$ (2) $\sqrt{\frac{15}{2}}$ (3*) $\frac{\sqrt{15}}{2}$ (4) $\sqrt{\frac{19}{2}}$

Sol. Let point at minimum distance from O is

$(h, h^2 - 4)$

$\therefore OP^2 = h^2 + (h^2 - 4) 2h = 0$

$\Rightarrow h = \pm\sqrt{\frac{7}{2}}, 0$

$\left(\frac{d^2(OP^2)}{dh^2}\right)_{h=\pm\sqrt{\frac{7}{2}}} > 0$

$\therefore$ OP is min at $h = \pm\sqrt{\frac{7}{2}}$

$OP_{min} = \sqrt{\frac{7}{2} + \left(\frac{7}{2} - 4\right)^2} = \frac{\sqrt{15}}{2}$

74. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$ = $(\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then A + B + C equals

(1) $\frac{21}{5}$ (2*) $\frac{16}{5}$ (3) $\frac{7}{10}$ (4) $\frac{27}{10}$

Sol. $I = \int \frac{dx}{\cos^3 + x\sin^{\frac{1}{2}} x\cos^{\frac{1}{2}} x} = \frac{1}{2}\int \frac{(\tan^2 x + 1)\sec^2 x}{(\tan x)^{\frac{1}{2}}}dx$

$I = \frac{1}{2}\int t^{\frac{3}{2}}dt + \frac{1}{2}\int t^{-\frac{1}{2}}dt$

Tanx = t

$I = \frac{1}{2}\int t^{\frac{3}{2}}dt + \frac{1}{2}\int t^{-\frac{1}{2}}dt$

$= \frac{t^{\frac{5}{2}}}{5} + t^{1/2} + c = \frac{(\tan x)^{\frac{5}{2}}}{5} + (\tan x)^{1/2}$

$A + B + C = \frac{16}{5}$

75. If $2\int_0^1 \tan^{-1} x dx = \int_0^1 \cot^{-1}(1 - x + x^2)dx$, then $\int_0^1 \tan^{-1}(1 - x + x^2)dx$ is equal to

(1) log 4 (2) $\frac{\pi}{2}$ + log 2 (3*) log 2 (4) $\frac{\pi}{2}$ – log 4

Sol. $2\int_0^1 \tan^{-1}x dx = \int_0^1 \cot^{-1}(1 - x + x^2)dx$ ....(1)

$\int_0^1 \tan^{-1}(1 - x + x^2)dx = \int_0^1 \left\{\frac{\pi}{2} - \cot^{-1}(1 - x + x^2)dx\right\} = \frac{\pi x}{2}\Big|_0^1 - 2\int_0^1 \tan^{-1} x dx = \frac{\pi}{2} - 2\left(\frac{\pi}{2} - \frac{1}{2}\ell n2\right) = \ell n2$

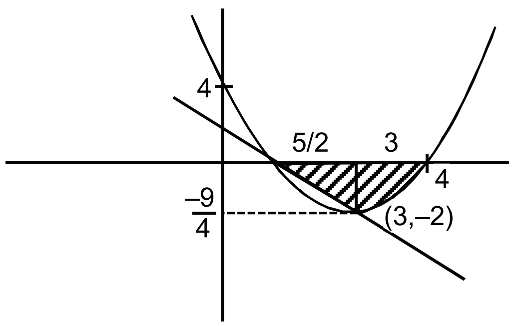
76. The area (in sq. units) of the region described by

A= {(x, y) | $y \geq x^2 - 5x + 4$, $x + y \geq 1$, $y \leq 0$} is

(1) $\frac{7}{2}$ (*2) $\frac{19}{6}$ (3) $\frac{13}{6}$ (4) $\frac{17}{6}$

Sol. A = {(x,y)|$y \geq x^2 - 5x + 4$, $x + y \geq 1$, $y \leq 0$}

Here $y \geq x2 - 5x + 4$, $x + y \geq 1$, $y \leq 0$



Required area $= \frac{1}{2}.22 + \int_3^4 (5x - x^2 - 4)dx$

$= 2 + \left[\frac{5x^2}{2} - \frac{x^2}{3} - 4x\right]_3^4$

$= 2 + \frac{5}{2}(16 - 9) - \frac{1}{3}(64 - 27) - 4(4 - 3)$

$= 2 + \frac{5}{2}(16 - 9) - \frac{1}{3}(64 - 27) - 4(4 - 3) - 2 + \frac{35}{2} - \frac{37}{2} - 4 = \frac{19}{6}$

77. If f (x) is a differentiable function in the interval $(0, \infty)$ such that f (1) = 1 and $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x} = 1$, for each x > 0, then $f\left(\frac{3}{2}\right)$ is equal to :

(1) $\frac{13}{6}$ (2) $\frac{23}{18}$ (3) $\frac{25}{9}$ (4*) $\frac{31}{18}$

Sol. Differentiate w.r.t. t

$$\lim_{x\to\infty}\frac{2t\,f(x)-x^2f'(t)=1}{1}=1$$

$\Rightarrow$ $2x\,f(x) - x^2f'(x) = 1$

$$f'(x)=\frac{2x\,f(x)-1}{x^2}$$

$$\frac{dy}{dx}=\frac{2y}{x}-\frac{1}{x^2}$$

$$\text{I.F.}=e^{-\int\frac{2}{x}dx}$$

$$=e^{-2\ell nx}=\frac{1}{x^2}$$

$$y\left(\frac{1}{x^2}\right)=\int-\frac{1}{x^4}dx$$

$$\frac{y}{x^2}=\frac{1}{3x^3}+c$$

at x = 1, y = 1

$\Rightarrow$ $c=\frac{2}{3}$

$$f(x)=\frac{1}{3x}+\frac{2x^2}{3}$$

$$f\left(\frac{3}{2}\right)=\frac{31}{18}$$

78. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=a$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is :

(1) 6xy = 7(x + y) (2) $4(x + y)^2 - 28(x + y) + 49 = 0$

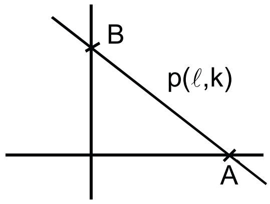
(3*) 7xy = 6(x + y) (4) $14(x + y)^2 - 97(x + y) + 168 = 0$

Sol. 4x + 3y = 12 ....(1)

3x + 4y = 12 ....(2)

Equation of lines passing through the intersection of the lines

$4x + 3y - 12 + \lambda(3x + 4y - 12) = 0$



$A = C\left(\frac{12(1+\lambda)}{4+3\lambda}, 0\right)$

$B = \left(0, \frac{12(1+\lambda)}{3+4\lambda}\right)$

$\ell n = \frac{6(1+\lambda)}{4+3\lambda}$ ....(3)

$k = \frac{6(1+\lambda)}{3+4\lambda}$ ....(4)

From (3) & (4)

$\lambda = \frac{3k-4h}{3h-4k}$ put in (1)

7hk = 6(h + k)

Hence locus is 7xy = 6(x + y)

79. The point (2, 1) is translated parallel to the line L : x – y = 4 by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is :

(1) $x + y = 2 - \sqrt{6}$ (2) $x + y = 3 - 3\sqrt{6}$ (3*) $x + y = 3 - 2\sqrt{6}$ (4) $2x + 2y = 1 - \sqrt{6}$

Sol. Slopes of x – y = 4

$\Rightarrow$ tanθ = 1 $\Rightarrow$ $\left(\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}\right)$

or $\left(\sin\theta = -\frac{1}{\sqrt{2}}, \cos\theta = -\frac{1}{\sqrt{2}}\right)$

Q is $\left(2 + 2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right), 1 + 2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right)\right)$

$\left(2-\sqrt{6}, 1-\sqrt{6}\right)$

Equation of required line is $x + y = 3 - 2\sqrt{6}$

80. A circle passes through (–2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle ?

(1) 4x + 5y – 6 = 0 (2*) 2x – 3y + 10 = 0 (3) 3x + 4y – 3 = 0 (4) 5x + 2y + 4 = 0

Sol. Required circle is

$(x - 0)^2 + (y - 2)^2 + \lambda(x) = 0$

It passes (–2, 4)

$\therefore$ 4 + 4 – 2λ = 0

λ = 4

$\therefore$ circle is $x^2 + y^2 - 4y + 4x + 4 = 0$

Centre (–2, 2) which satisfy

2x – 3y + 10 = 0 Ans 3.

81. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2 - 18e + 5 = 0$. If S(5, 0) is a focus and 5x = 9 is the corresponding directrix of this hyperbola, then $a^2 - b^2$ is equal to :

(1) 7 (2*) –7 (3) 5 (4) –5

Sol. $9e^2 - 18e + 5 = 0$

$\Rightarrow e = \frac{5}{3}$

$\therefore 1 + \frac{b^2}{a^2} = e^2 = \frac{25}{9}$

Also distance between foci and directrix is

$= \left(ae - \frac{a}{e}\right) = 5 - \frac{9}{5}$

$\Rightarrow a\left(\frac{5}{3} - \frac{3}{5}\right) = \frac{16}{5} \Rightarrow a = 3$

Form (i)

$1 + \frac{b^2}{9} = e^2 = \frac{25}{9} \Rightarrow b^2 = 16$

$\therefore a^2 - b^2 = 9 - 16 = -7$

82. If the tangent at a point on the ellipse $\frac{x^2}{27} + \frac{y^2}{3} = 1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is :

(1) $\frac{9}{2}$ (2) $3\sqrt{3}$ (3) $9\sqrt{3}$ (4*) 9

Sol. Let P $\left(3\sqrt{3}\cos\theta, \sqrt{3}\sin\theta\right)$

$\therefore$ tangen is $\frac{x}{3\sqrt{3}}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$

$\Rightarrow A\left(3\sqrt{3}\sec\theta, 0\right) \quad B\left(0, \sqrt{3}\operatorname{cosec}\theta\right)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}OA.\ OB$

$= \frac{1}{2}\left(3\sqrt{3}\sec\theta.\quad \sqrt{3}\operatorname{cosec}\theta\right)$

$= \frac{9}{2\sin\theta\cos\theta} = \frac{9}{\sin 2\theta}$

$\therefore$ minimum area of $\Delta OAB = \frac{9}{1} = 9$

83. The shortest distance between the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{x-5}{4}$ lies in the interval :

(1) [0, 1) (2) [1, 2) (3*) (2, 3] (4) (3, 4]

Sol. $\frac{x}{2} = \frac{y}{z}\frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

Shortest distance

$= (\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)$

here $\vec{b}_1 - \vec{b}_2 = (2i + 2j + k) \times (-i + 8j + 4k)$

= – 9j + 18k

$(\vec{b}_1 \times \vec{b}_2) = \frac{-j + 2k}{\sqrt{5}}$

$\vec{a}_2 - \vec{a}_1 = -2i + 4j + 5k$

$\therefore$ S.D.$(-2i + 4j + 6k).\frac{(-j + 2k)}{\sqrt{5}} = \frac{6}{\sqrt{5}}$ which lies in $(2,3)$

84. The distance of the point (1, –2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes x – y + 2z = 3 and 2x – 2y + z + 12 = 0, is :

(1*) $2\sqrt{2}$ (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol. Equation of plane $\perp$ to the planes.

x – y + 2z = 3 & 2x – 2y + z + 12 = 0

and passes through (1, 2, 2) is

$$\begin{vmatrix} x-1 & y-2 & z-2 \\ 1 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 0$$

3(x – 1) + 3(y – 2) = 0

x + y = 3 ..... (1)

distance of plane x + y – 3 = 0 from (1, – 2, 4) is

$$\left|\frac{1-2-3}{\sqrt{1+1}}\right| = 2\sqrt{2}$$

85. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i} + \hat{j} - \hat{k}$, $-\hat{i} + 3\hat{j} + p\hat{k}$ and $5\hat{i} + 9\hat{j} - 4\hat{k}$, then the point (p, q) lies on a line :

(1) parallel to x-axis.

(2) parallel to y-axis.

(3*) making an acute angle with the positive direction of x-axis.

(4) making an obtuse angle with the positive direction of x-axis.

Sol. $\overline{AB} = -4i + 2j + (p+1)\hat{k}$

$\overline{AC} = -2i + (q-1)\hat{j} - 3\hat{k}$

$\overline{AB}.\overline{AC} = 0 \quad \Rightarrow \quad -8 + 2(9-1) - 3(p+1) = 0 \quad \Rightarrow \quad -3p + 2q - 13 = 0$

$\Rightarrow$ (p, q) lies on line

$3x - 2y + 13 = 0$

$\text{slope} = \frac{3}{2}$

86. If the mean deviation of the numbers 1, 1 + d, ..., 1 + 100d from their mean is 255, then a value of d is :

(1*) 10.1 (2) 20.2 (3) 10 (4) 5.05

Sol. Mean is $\frac{101 + \frac{100 \times 101}{2}}{101} = 1 + 50\text{ d}$

Sum of deviation about mean is

50 d + 49d +..........d + 0 +......+ 50 d

= 50.51 d

$\text{Mean deviation} = \frac{50 \times 51d}{101} = 255$

$d = \frac{255 \times 101}{2550} = 10.1$

87. If A and B are any two events such that P(A) = $\frac{2}{5}$ and P(A ∩ B) = $\frac{3}{20}$, then the conditional probability, P(A | (A' ∪ B')), where A' denotes the complement of A, is equal to :

(1) $\frac{1}{4}$ (2*) $\frac{5}{17}$ (3) $\frac{8}{17}$ (4) $\frac{11}{20}$

Sol. $P(A) = \frac{2}{5}$

$P(A \cap B) = \frac{3}{20}$

P(A/A' ∪ B')) = ?

$$P(A/(A' \cup B')) = \frac{P(A \cap (A' \cup B'))}{P(A' \cap B')} = \frac{P((A \cap A') \cup (A \cap B'))}{P(A \cap B)'} = \frac{P(\phi \cup (A \cap B'))}{1 - P(A \cap B)}$$

$$= \frac{P(A \cap B')}{1 - \frac{3}{20}} = \frac{P(A) - p(A \cap B)}{\frac{17}{20}} = \frac{\frac{2}{5} - \frac{3}{20}}{\frac{17}{20}} = \frac{\frac{5}{20}}{\frac{17}{20}} = \frac{5}{17}$$

88. The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is

(1) 2 (2) 4 (3) 6 (4*) 8

Sol. $2\sin^4 x + 18\cos^2 x = 1 + 2\cos^4 x + 18\sin^2 x + 2\sqrt{2\cos^4 x + 18\sin^2 x}$

$2(\sin^2 x - \cos^2 x) + 18(\cos^2 x - \sin^2 x) = 1 + 2\sqrt{2\cos^4 x + 18\sin^2 x}$

$\Rightarrow \quad 16(\cos^2 x - \sin^2 x) = 1 + 2\sqrt{2\cos^4 x + 18\sin^2 x}$

$\Rightarrow \quad 16\cos 2x - 1 = 2\sqrt{2\left(\frac{1 + \cos 2x}{2}\right)^2 + 9(1 - \cos 2x)}$

$\Rightarrow$ $256\cos^2 2x + 1 - 32\cos 2x = 4\left(\frac{1+2\cos 2x+\cos^2 2x}{2} + 9(1-\cos 2x)\right)$

$\Rightarrow$ $256\cos^2 2x + 1 - 32\cos 2x = 2(19 - 16\cos 2x + \cos^2 2x)$

$\Rightarrow$ $254\cos^2 2x = 37$

$\Rightarrow$ $\cos^2 2x = \frac{37}{254}$ $\Rightarrow$ $\cos 2x = \pm\sqrt{\frac{37}{254}} \in [-1, 1]$

clearly 8 solutions

89. If m and M are the minimum and the maximum values of $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$, $x \in R$, then M – m is equal to :

(1) $\frac{15}{4}$ (2*) $\frac{9}{4}$ (3) $\frac{7}{4}$ (4) $\frac{1}{4}$

Sol. $4 + \frac{1}{2}\sin^2 2x - \frac{1}{2}(2\cos^2 x)^2$

$= 4 + \frac{1}{2}\sin^2 2x - \frac{1}{2}(1+\cos 2x)^2 = -\cos^2 2x - \cos 2x + 4 = -[\cos^2 2x + \cos 2x - 4] = \frac{17}{4} - \left(\cos 2x + \frac{1}{2}\right)^2$

M = maximum value = $\frac{17}{4}$

M = minimum value = 2

$M - m = \frac{17}{4} - 2 = \frac{9}{4}$.

90. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals :

(1) (T, T) (2) (T, F) (3*) (F, T) (4) (F, F)

Sol. Statement P is False

Statement Q is True.

$V_1 \equiv F$

$V_2 \equiv T$

Ans. 1